

4.1 Extreme values of functions

local vs global extreme values

if f has a max/min then f' might be 0 or f' might be undefined but for sure, f' changes sign

critical points, endpoints candidate list for extrema

$f' = 0$
or $f' \neq$

Oct 18-8:47 AM

find extrema of $f(x)$ on $[-2, 3]$. Solve graphically and analytically

max, min

Define $f(x) = x^{-\frac{1}{3}}$

$f' = \frac{2}{3}x^{-\frac{4}{3}} = 0$
 $f' = \frac{2}{3\sqrt[3]{x}} = 0$

what $f(0) = 0 = \min$
 $f(-2) = (-2)^{-\frac{1}{3}} = \sqrt[3]{-4}$
 $f(3) = 3^{-\frac{1}{3}} = \sqrt[3]{9} = \max$

candidate list (where)

- $f' = 0$ never min at $x=0$
- $f' \neq$ $x=0$
- endpoints $x=-2$
 $x=3$ max at $x=3$

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find extrema of $f(x)$. Solve graphically and analytically

Define $f(x) = \frac{1}{\sqrt{4-x^2}} = (4-x^2)^{-1/2}$

$f' = (-2x)(-\frac{1}{2})(4-x^2)^{-3/2}$
 $f' = \frac{x}{\sqrt{(4-x^2)^3}}$

$\lim_{x \rightarrow 2} \frac{1}{\sqrt{4-x^2}} = \infty$

stop?

candidate list plug in f

$f' = 0$ at $x=0$
 $f' \neq$ at $x=2$ is $y = \frac{1}{2}$
 $x=2$

endpoints - none
domain $(-2, 2)$

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Define $f(x) = \begin{cases} 5-2x^2, & x \leq 1 \\ x+2, & x > 1 \end{cases}$

$f'(x) = \begin{cases} -4x & x < 1 \\ 1 & x > 1 \end{cases}$

lh deriv. \neq rh deriv at $x=1$

candidate list where

$f' = 0$ at $x=0$
 $f' \neq$ at $x=1$ what local max
 $f(0) = 5$
 $f(1) = 3$ local min

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Define $f(x) = \ln\left(\frac{x}{1+x^2}\right)$ Done

$f' = \frac{1}{\left(\frac{x}{1+x^2}\right)} \cdot \frac{(1+x^2)1 - x \cdot 2x}{(1+x^2)^2}$

$f' = \frac{1-x^2}{x(1+x^2)}$

$f' = 0 \quad x = 1$
 $x = -1$

~~$f' = x \quad x = 0$~~
 $x = 0$ not in domain

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